A Shape Factor Method for Computing Heating Loads from Building Slab-on-Grade Foundations

W.W. Rust, Ph.D., P.E.
Member ASHRAE

J. E. Christian
Member ASHRAE

ABSTRACT

A shape factor method, designated the Z-shape factor, is presented, which provides a simple, hand calculation technique of predicting daily, weekly, or seasonal heat loads due to the effects of slab-on-grade earth coupling. This semi-analytic method accommodates a wide range of time periods and local soil and weather conditions and is limited to vertical insulation schemes. The unique feature of the Z-shape factor method is that it allows a user to input soil conductivity and several site-specific geometric parameters. This extends the usefulness of the general shape factor method available in the ASHRAE Handbook *and other recent publications since more site-specific inputs can be utilized in the below-grade heat loss estimation. Predictions derived from the shape factor method are compared to other simplified predictive procedures and to field data. The field data collected from an earth-coupling experiment being conducted at a laboratory are used to check the predicted heat loss by the Z-shape factor method from both an insulated and uninsulated slab-on-grade. Results are within 10% of the alternative computations and of the field data, which suggests the procedure is suitable for most annual heat and peak heating load calculations for slab-on-grade structures.*

INTRODUCTION

It is estimated that the United States' earth-coupled heat loss is between 1 and 3 quadrillion kilojoules (0.9 to 2.7 quads) annually in the United States (Claridge 1988). As building designers achieve greater efficiency in above-grade building components, the effects of earth coupling can constitute a large fraction of the total loss. Therefore, it is important to understand below-grade heat loss in order to design foundation insulation systems with the same cost-effectiveness as the other energy-efficient features of the building.

There are numerous methods of computing below-grade heat loss available in the literature that vary in complexity, applicability, and utility. Most simplified algorithms for estimating the heat loss from a slab-on-grade foundation have the basic structure

$$Q/L = (T_1 - T_2)/(R + ER) \tag{1}$$

where

Q/L = heat loss per unit of perimeter length (L) per unit of time,
R = thermal resistance of the slab plus air film,
ER = effective resistance of the soil,
T_1 = inside air temperature, and
T_2 = outside air temperature.

The combined term $(R + ER)$ may be thought of as the overall resistance of the slab-earth system or the system resistance; its reciprocal $(R + ER)^{-1}$ is the overall heat transfer coefficient or "U-value" for the system. In a slab-on-grade, the heat loss equation is sometimes written

$$Q = kS\,(T_1 - T_2) \tag{2}$$

where

S = dimensionless shape factor for the system,
k = thermal conductivity of the soil.

Equation 1 can be set equal to Equation 2 and solved for S so that

$$S = L/k\,(R + ER). \tag{3}$$

The "edge factor" method combines the terms k and S into a single edge factor, F, which is the heat transfer coefficient or U-value:

$$F = U = kS = L/(R + ER). \tag{4}$$

The Z-shape factor method presented here has been developed as a semi-analytical tool that is an outgrowth of detailed numerical solutions to specific, idealized slab heat transfer calculations (Rust 1991). The intent of this method is to provide a fast and reasonably accurate scheme that is sufficiently general to be applied to a variety of slab foundations. When compared to other computational methods, such as the interzonal temperature profile estimation (ITPE) numerical method (Krarti and Claridge 1988; Krarti et al. 1990), the seasonally averaged shape factor results agreed to within 10% for reported calculations (Rust 1991).

THE Z-SHAPE FACTOR METHOD

The essence of the Z-shape factor method is that it relates the nondimensional heat loss of the slab (similar to a Nusselt number) to local conditions (indoor and outdoor

William W. Rust is with the Oak Ridge Associated Universities on a postdoctoral fellowship, and Jeffrey E. Christian is the leader of the Building Thermal Envelope Systems and Materials Program at Oak Ridge National Laboratory, Oak Ridge, Tennessee.

seasonal temperatures, average outdoor annual temperature, and soil thermal conductivity) by employing heat transfer parameters (U-values and Biot numbers). These relationships were originally found by parametric studies of idealized two-dimensional slabs (thin plane with zero thickness), but further research indicates that real slab heat loss may also be computed to a good approximation by modeling the real slab with the same heat transfer parameters. The method may be extended to three dimensions by decomposition and superposition. The three-dimensional slab-on-grade heat loss calculation is decomposed into two quasi-two-dimensional calculations and worked separately. The final result is determined by superposing the two intermediate solutions by simple addition.

Figure 1 shows a typical slab construction of dimensions (a x b). To analyze this heat loss for a specific climate from first principles is involved and time consuming. There are other approximate methods that may be used as well; however, the Z-shape factor for uninsulated slabs and those insulated with vertical perimeter insulation is a more exact estimate of heat loss generated by several simple equations incorporating important site-specific variables.

EXAMPLE OF A DESIGN HEAT LOSS ESTIMATION FOR AN UNINSULATED SLAB

The following example shows the Z-shape factor method applied to an uninsulated slab-on-grade. First is a listing of needed input data and then a seven-step solution leading to an estimate of the total slab heat loss.

Step 1 Calculate or estimate characterization inputs. The slab has dimensions (a x b), as depicted in Figure 1. The Z-shape factor method is used to find the heat loss to the ambient with the following inputs:

- average indoor air temperature over the slab for period T_1,
- average ambient air temperature for period T_2,
- annual average ambient air temperature, T_3, and
- depth below grade where soil temperature coincides with T_3, D.

In addition, the soil thermal conductivity, vertical insulation depth (if present), physical thickness, thermal conductivity of the concrete, and other related physical dimensions of the slab and insulation must be known or estimated.

Step 2 Select a direction and compute the heat transfer coefficients and effective soil thermal conductivity for that direction. The required values are

- soil thermal conductivity, k (composite value if soil is composed of distinctly different regions);
- slab U-value (including interior floor film coefficient), U_1; and

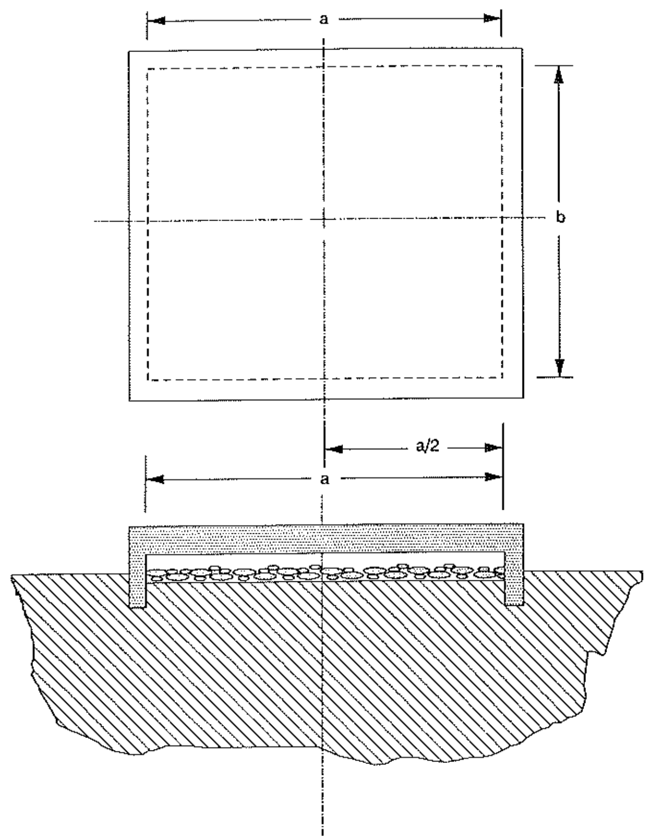

Figure 1 *Typical slab-on-grade with (a x b) dimensions.*

- outdoor U-value (includes exterior soil top surface film coefficient and any pavement covering), U_2.

By excluding the soil, the U-values are computed. If there is no surface covering the soil adjacent to the building, such as a concrete or asphalt drive, then the outdoor surface film coefficient is equal to the outdoor U-value:

$$U_i = (1/h_i + \sum 1/R_{thj})^{-1} \tag{5}$$

where

h = heat transfer film coefficient,
i = 1 or 2 for the indoor or outdoor U-value, and
R_{thj} = R-value of the layers of insulating materials between the foundation and outdoor surface covering.

If the soil is homogeneous, then the thermal conductivity needs no correction. However, if heat flows through regions of distinctly different composition, then this may be taken into account by estimating a composite thermal conductivity. If this is done, overall accuracy is improved, especially in the slab edge region. The path for heat flow from an uninsulated foundation can be imagined as a series of individual, similarly shaped strips that are practically semicircular. Heat flows through the series path composed of fractions α and β of material in direct proportion to each path length, as shown in Figure 2. The composite thermal conductivity is

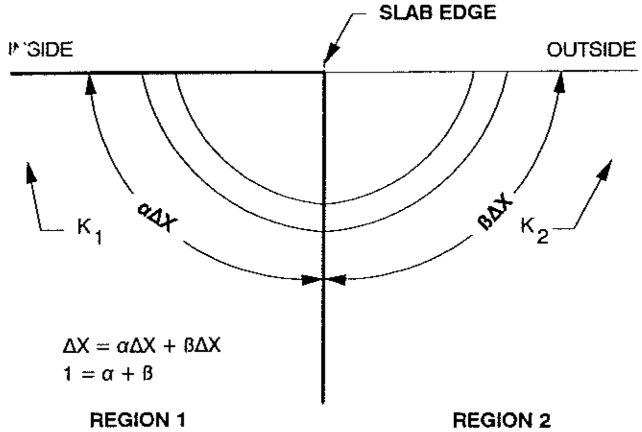

Figure 2 *Estimating apparent thermal conductivity for regions of different composition.*

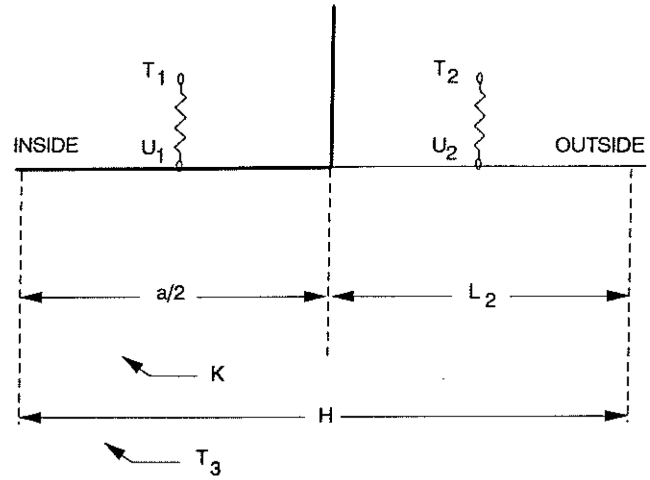

Figure 3 *Slab equivalent thermal model.*

$$k = 1/(\alpha/k_\alpha + \beta/k_\beta) \tag{6}$$

where

$$\alpha + \beta = 1.$$

For paths of equal length, then

$$k = 1/(\alpha/k_\alpha + \beta/k_\beta). \tag{7}$$

Find the following nondimensional parameters in steps three through six:

Step 3 Calculate the Biot number for the half-slab *(a/2)*:

$$Bi = U_1(a/2)/k. \tag{8}$$

Step 4 Estimate the following quantity:

$$r = U_1\ (a/2)/U_2L_2. \tag{9}$$

The quantity r is designated as the cooling factor and represents the cooling effect of the outdoor conditions on the slab surface where U_1 and U_2 are the respective indoor and outdoor U-values, $a/2$ is the slab half-length, and L_2 is the flux horizontal penetration distance from slab edge. Figure 3 indicates the relationship of these quantities to the slab. The distance H is the total flux penetration distance out from the centerline of the slab. Figure 4 is used to determine H^* once G^*, the ground flux parameter, is calculated.

At this point, all the values are known except L_2, which is found as follows:

Defining the following terms to be

$$H^* = H/a, \tag{10}$$

$$G^* = \pi\ (a/D)(T_3 - T_2)/(T_1 - T_2), \tag{11}$$

where H^* = horizontal flux penetration ratio and G^* = ground flux parameter,

$$L_2 = H - (a/2) = a\ H^* - (a/2). \tag{12}$$

These flux parameters are based on Carslow and Jaeger's identification of a radial geothermal gradient (Rust 1991). Substituting into Equation 9, the expression for r becomes

$$r = U_1/[U_2(2H^* - 1)]. \tag{13}$$

Next, G^* is determined by using Equation 11. Figure 4 (Rust 1991) is then used to determine H^* and is substituted into Equation 13 to find r.

Step 5 The Z-shape factor is given by the following numerically derived relation (Rust 1991):

$$Z = 3.3654 + 2.6102(1 + r)/Bi. \tag{14}$$

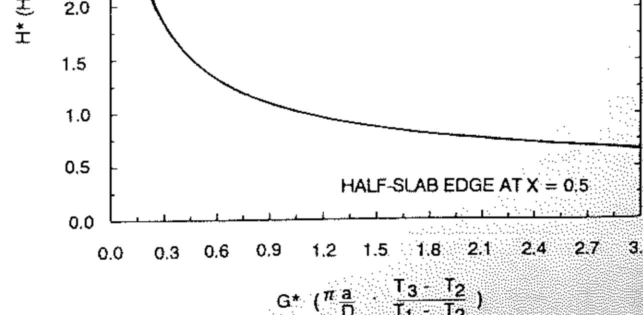

Figure 4 *Horizontal penetration ratio (H*) vs. ground flux parameter (G*).*

Step 6 The nondimensional total heat flux for the half-slab is given by the following relation:

$$S = Q/bk(T_1 - T_2) = \ln[1 + Z * Bi/(1 + r)]/Z. \quad (15)$$

Equations 14 and 15 are results of heat loss parametric studies using a detailed finite-difference model of an "ideal slab," which is defined as having the same thermophysical properties as the adjacent soil.

Step 7 The total heat flux for the slab in the a direction is Q_a. Then,

$$Q_a = 2bk(T_1 - T_2)S, \quad (16)$$

$$\frac{Q_a}{b} = 2k(T_1 - T_2)S.$$

Step 8 Steps 1 through 6 are repeated when the b direction is taken as the half-slab length and a and b results are added. This procedure ignores the corner effects. The total slab heat loss in three dimensions is

$$Q = Q_a + Q_b. \quad (17)$$

THE Z-SHAPE FACTOR APPLIED TO VERTICALLY INSULATED SLABS-ON-GRADE

In most climates it is cost-effective to use slab edge insulation, which reduces the quantity of heat leaving the slab. Good design practices should include a determination of the most effective level of insulation. The Z-shape factor method can be used to analyze the slab-on-grade heat loss and determine the effective vertical insulation thickness and depth. Perimeter insulation placed horizontally underneath a slab cannot be evaluated with this methodology.

For this example, consider a slab in two dimensions for simplicity. If a perfect insulating material were inserted at both edges to a depth c, the path for heat transmission would be strictly into the earth and back up to the outside surface. Imagine a subterranean plane parallel to the earth surface at a depth c. The material between this subterranean plane and the earth's surface is an additional distance through which the slab heat is required to pass as it moves down into the earth, around the insulation, and back out to the surface. The earth above this subterranean plane acts as an insulating material. The heat transfer U-values are modified to include this additional earth material, so U_1 becomes

$$U_1 = (1/U_{slab} + c/k)^{-1} \quad (18)$$

and U_2 becomes

$$U_2 = (1/U_{surface} + c/k)^{-1}. \quad (19)$$

These values may be used to compute the slab heat loss for the case of perfect vertical insulation using the procedure (Steps 1 to 8) in the previous section. This is clearly the best result that could be obtained by any vertical edge insulation system.

Unfortunately, there is no "perfect" insulation. A portion of the heat exits through the earth around the insulation and a portion exits through the vertical insulation. In reality, heat flows in two dimensions; however, as an approximation, this procedure assumes the earth under the slab and in between the slab edge insulation is a thick "fin." Secondly, this procedure assumes one-dimensional heat flow by making the temperature at any given depth in this earth volume to be uniform. Results from numerical simulations of this geometry support that this assumption is approximately true to within about 1°C to 2°C (2°F to 4°F). This procedure ignores the dynamic effects of the soil thermal mass storage, which suggests this will work all right for shallow foundations, such as slabs, but not for deep basements, particularly at the beginning and end of the heating season. This approach assumes that the cold side of the insulation is the same as the ambient outdoor air. This assumption results in a maximum likely heat loss.

Consider the earth volume depicted in Figure 5 as bounded on top by the slab surface, the side by the outer surface of the slab edge insulation, and the bottom perimeter coinciding with the subterranean plane. The heat balance for the entire volume is

$$Q_1 = Q_2 + Q_I \quad (20)$$

where Q_1 is the total heat flow transmitted into the slab, Q_2 is the total heat flow exiting the volume at the subterranean plane, and Q_I is the total heat flow leaving the volume through the slab edge insulation.

The heat balance equation on a horizontal differential strip yields

$$\theta'' - \gamma^2\theta = 0 \quad (21)$$

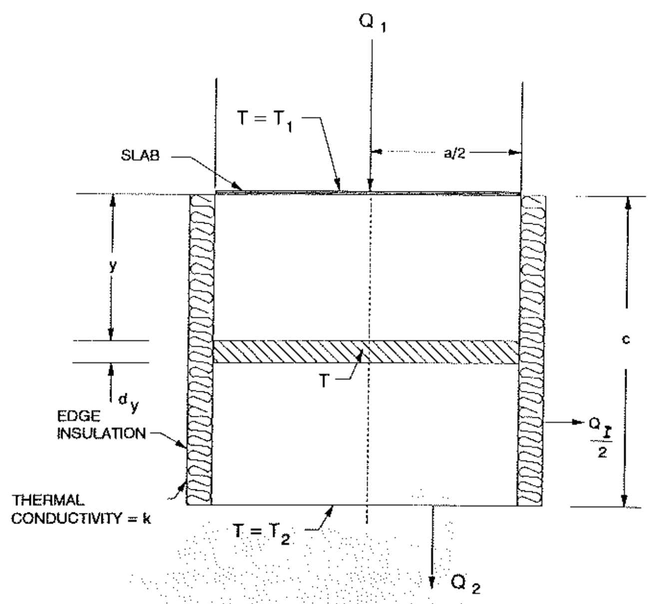

***Figure 5** One-dimensional slab heat loss sketch.*

where

$$\Theta = (T - T_2)/(T_1 - T_2),$$

$$\Theta'' = \frac{\partial^2 \Theta}{\partial Y^2},$$

$$Y = y/c,$$

c = insulation depth,
y = specific depth for heat flux of interest,

$$0 \leq y \leq c,$$

and

$$\gamma^2 = \frac{2k_I c^2}{k\ a\ b}, \tag{22}$$

$$\frac{\partial \theta}{\partial Y}]_{Y=0} = \Theta'_1 = - \frac{Q_1 c}{k\ a\ b(T_1 - T_2)} \quad at\ Y{=}0,$$

$$\frac{\partial \theta}{\partial Y}]_{Y=1} = \Theta'_2 = - \frac{Q_2 c}{k\ a\ b(T_1 - T_2)} \quad at\ Y{=}1.$$

The general solution to Equation 21 is

$$\Theta = C_1\ e^{-\gamma Y} + C_2\ e^{\gamma Y}. \tag{23}$$

The first boundary condition gives

$$-C_1 + C_2 = \frac{1}{\gamma}\ \Theta'_1.$$

The second boundary condition gives

$$-C_1\ e^{-\gamma} + C_2\ e^{\gamma} = \frac{1}{\gamma}\ \Theta'_2.$$

The solution remains finite: $C_2 = 0$ and

$$C_1 = -\frac{1}{\gamma}\ \Theta'_1. \tag{24}$$

The particular solution to Equation 21 with the previous boundary conditions is

$$\Theta = -\frac{1}{\gamma}\ \Theta'_1\ e^{-\gamma Y}. \tag{25}$$

Integrating the heat flux Θ'_I for $0 \leq Y \leq 1$ along the insulation yields

$$Q_I^* = S(1 - e^{-\gamma}) \tag{26}$$

where

$$Q_I^* = \frac{Q_I}{k_I b(T_1 - T_2)}, \tag{27}$$

Q_I = total heat loss through the vertical insulation, and
S = nondimensional heat flux.

Therefore, to obtain the average heat loss for a given period, one multiplies Q_I* by k and ΔT.

THE Z-SHAPE FACTOR AND FIELD MEASUREMENT COMPARISON

Thermal data are being collected for evaluation of the slab-on-grade foundation supporting the experimental facility called the roof thermal research apparatus (RTRA) located at a national laboratory. The building subgrade is instrumented to monitor thermal exchanges between the building and the environment. The foundation area is monitored automatically, and hourly averaged data are stored for retrieval and analysis. Thermocouples and heat flux transducers are placed strategically to detect changes in the temperature field in the earth and slab foundation as well as to measure the flow of heat at selected depths near the vertical foundation wall, as shown in Figures 6 and 7.

Mathematical models are used to predict the measured change of heat flux and the temperature field. A full-scale, real-time simulation is used to validate the physics in the mathematical modeling by comparing the simulation predictions to the measured response. The impact of water on the flow of heat in a porous media can be substantial. Furthermore, isolating the major effects of property changes caused by the hysteresis of water movement in a media,

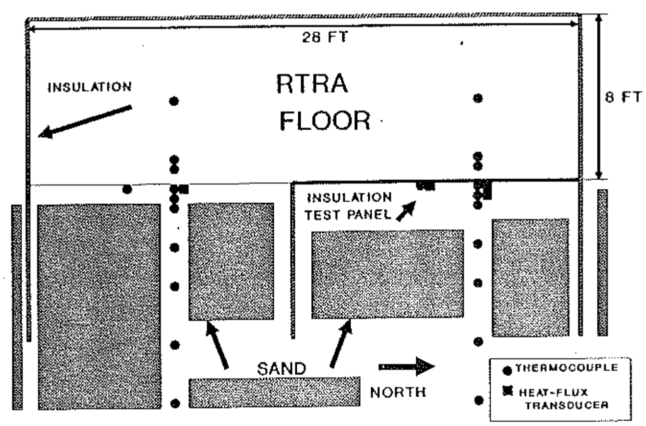

Figure 6 *Sensor location on RTRA floor plan for slab-on-grade insulation test.*

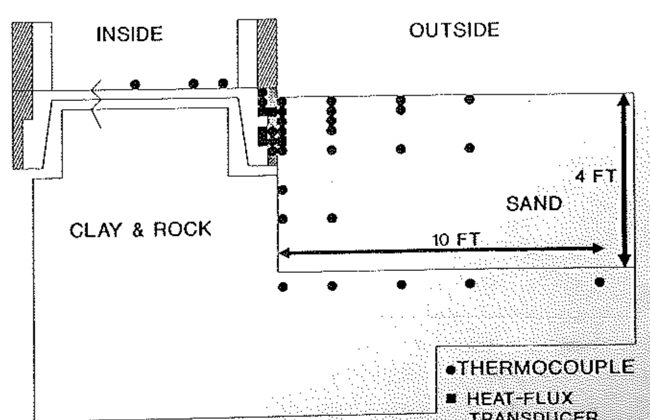

Figure 7 *Instrument plane through insulated slab cross section.*

such as in-situ foundation soil, is extremely difficult. Therefore, it is important to understand the limits of accuracy of the particular modeling process employed. For this RTRA modeling, the heat equation has been solved by employing an alternating direction implicit (ADI) scheme that is second-order accurate in space and time (Anderson et al. 1984). This was performed using constant soil density, specific heat, and thermal conductivity and was compared to the results in which these soil properties varied as a function of space and time in consonance with the outdoor conditions. The heat equation is

$$\rho c \frac{\partial T}{\partial t} = \bar{\nabla} \cdot (k \bar{\nabla} T). \tag{28}$$

Boundary conditions in the field are estimated by inspecting the actual isothermal plots from the data. It was found that agreement could be obtained for periods of relatively dry weather by using adiabatic vertical boundaries. The interior and exterior surface film coefficients were chosen from within the range of typical values to best emulate the average conditions observed during the time period December 3, 1990, to February 4, 1991. Seasonal average indoor temperature and actual outdoor hourly temperatures were used as the driving conditions for the transient problem.

The lower boundary condition was selected based on comparison of the measured flux and temperature fields with the computed values for both a constant temperature and constant flux condition. The flux computations were relatively insensitive to the boundary selected, but the best matching of the temperature field was produced by the constant flux boundary condition.

Using typical published thermal conductivities for clay and sand, the heat flux comparisons with measured results were wholly inadequate to describe the process, particularly during periods of moderate to heavy rain.

This inadequacy illustrated the need to measure the subgrade thermal conductivity of sand and clay. Thermal needles were installed at four locations in the sand and clay and were used to measure thermal conductivity. The needle consists of a metal tube, an electric resistance heater wire, and an axially positioned thermocouple junction located midway along the probe length. Power is applied to the resistance heater wire, and the temperature change with time is recorded. Using the linear portion of the time-temperature profile, the thermal conductivity of the surrounding soil can be calculated.

These measurements readily demonstrated the impact of locally heavy rain on the sand and, to a lesser extent, on clay. Using these data as a guide for the thermal conductivities of sand and clay, the agreement between the solution of the transient problem and measured data improved, especially for weekly periods of little or no rainfall.

The predicted transient thermal field obtained with the model is compared with data from the RTRA collected between December 3, 1990, and February 3, 1991. This heating period was chosen as a rigorous test for the numerical model because of the wide swings of temperature and rainfall. A detailed simulation of this period demonstrated the need for a coincident history of soil thermal conductivity. Soil conductivity data were collected from September 1991 to March 1992. This soil conductivity data were used for the two heating seasons. The seasons are quite similar and no serious penalty was incurred by correcting and overlaying soil conductivity data from one season to the other. By doing this, sufficient data exist to permit a Z-factor comparison of nondimensional and dimensional earth-coupled heat loads on the building for several periods.

Consider the following two-dimensional slab-on-grade heat loss: a 3 × 8.5 m (8 × 28 ft) building located in Tennessee that is (a) uninsulated and (b) has vertical insulation with a rated R-value of 1.85 m^2K/W @ 24°C (R-10.5 @ 75°F). The actual insulation system has 3.7 m^2K/W (R-21) for the top 0.25 m (10 in.) of which 0.15 m (6 in.) is above grade and 1.85 m^2K/W (R-10.5) for the bottom 2.5 m^2K/W (14 in.) of the foundation wall. Using the following averaged data (extracted or inferred from measurements), the heat loss by the Z-shape factor method was calculated and compared to field-measured heat loss.

Step 1 Calculate or estimate characterization inputs:

Test period: January 6, 1992 - February 3, 1992.

Ground temperature (T_3): 14.4°C (58°F) at 2.4 m (8 ft).

Indoor temperature (T_1): 22.8°C (73.2°F) (average for period 6 in. above floor and 6 in. from wall above uninsulated test wall).

Outdoor temperature (T_2): 2.92°C (37.3°F).

Indoor film coefficient (h_1): 8.0 W/m^2K (1.42 Btuh/ft^2/°F).

Average uninsulated floor surface temperature at 0.15, 0.3, and 1.2 m (0.5, 1.0, and 4.0 ft) from wall: 19°C (66.3°F), 20°C (68.1°F), and 22°C (71.5°F).

Average insulated floor surface temperature at 0.15, 0.3, and 1.2 m (0.5, 1.0, and 4.0 ft) from wall: 21.1°C (69.8°F), 21.4°C (70.5°F), and 23.2°C (73.7°F).

Step 2 Select direction, and compute U-values and soil conductivity:

Half-slab dimension: 1.22 m (4 ft).

Apparent soil thermal conductivity (k): 1.81 W/m^2K (1.0 Btu/ft°F). This is both a time- and space-averaged number using a two-month period (December 1991 and January 1992).

Indoor U-value (U_1): 4.28 W/m^2K (.75 Btu/ft^2h°F).

Outdoor U-value (U_2): 12.87 W/m^2K (2.27 Btu/ft^2h-°F).

Z-SHAPE FACTOR METHOD (UNINSULATED SIDE)

Step 3 Compute the inside Biot number:

Bi = 4.28 W/m^2K (1.22 m)/(1.81 W/mK).

Bi = 2.88.

Step 4 Compute the cooling factor *r*:

$G^* = \pi$ (8 ft/8 ft) (58 - 37.25) °F/ (73.2 - 37.25) °F.

$G^* = 1.81$.

Using Figure 4, the value of H^* is found for $G^* = 1.81$, which is $H^* = 0.83$.

r = 4.28[12.87(1.66 - 1)].

r = 0.504.

Step 5 Compute Z using Equation 14:

Z = 3.3654 + 2.6102(1+0.504)/2.88.

Z = 4.73.

Step 6 Compute the nondimensional heat load on the RTRA (*S*) using Equation 15:

S = ln [1 + (4.73) (2.88)/(1 + 0.504)]/4.73.

S = 0.488.

Step 7 Compute the half-slab dimensional heat load on the RTRA using Equation 16:

Q = (1.81 W/mK) x 22.8 - 2.92 (°C) (0.488).

Q = 17.56 W/m (18.36 Btuh/ft) for period.

EXPERIMENTAL EVIDENCE FOR RTRA HEAT LOSS FROM UNINSULATED SLAB

The only field data used to estimate the half-slab heat loss were from the surface temperatures and air temperature reading right above the floor. Using a surface film coefficient is a poor substitute for a heat flux meter, but that is all that is available for predicting heat flux at this location. The heat flux crossing the half-slab from the inside air may be expressed as

$$Q = h_1 \; a/2 \; (T_1 - T_{average\ surface}).$$

The average surface temperature is computed by assuming a quadratic profile for the floor temperature and integrating across the floor to find the quantity $a/2$ $(T_1 - T_{average\ surface})$ for the above data from January 6, 1992, to February 3, 1992. Three thermocouples located at the midpoint of the floor, 12 inches in from the slab edge and 6 inches in from the slab edge, are used to develop the integrated $T_{average\ surface}$. The film coefficient, *h*, was derived from a best fit of the period from December 3, 1990, to February 4, 1991.

Q = (8.0 W/m²K)* (2.18 mK)

Q = 17.48 W/m (18.33 Btuh/ft)

The nondimensional experimental heat flow $(q/k \, \Delta \, T)$ or shape factor for the period from January 6, 1992, to February 3, 1992, is 0.487. Repeating the calculations for other periods results in measured nondimensional heat flows on the uninsulated slab of the RTRA, shown in Table 1 under the column labeled "Data." The Z-shape factor model is also used to derive a set of comparable nondimensional heat flux values. The 30% difference between the data and the shape factor method prediction for the first period listed is believed to be caused by the fact that the entire slab edge was exposed to the ambient while instrumentation was being installed for the month of January. Thus, temperature conditions had not stabilized. For the last three heating seasons (1989-90, 1990-91, and 1991-92), the model predictions are within 10% of the measured response. Table 1 also contains the temperature difference between the RTRA inside measured air and the outdoor ambient dry-bulb for each heating period.

TABLE 1
RTRA Uninsulated Slab Data Compared to Z-Shape Factor Prediction

	ΔT	S		
	°C (°F)	Data*	Z-Shape factor	% Difference
February 14, 1989 - April 17, 1989	12.72 (22.90)	.7047	.4940	-30%
October 30, 1989 - April 8, 1990	14.54 (26.17)	.5457	.4886	-10.5%
November 11, 1990 - March 24, 1991	17.1 (30.73)	.5045	.5033	-0.2%
December 2, 1991 - March 9, 1992	17.1 (30.74)	.5218	.4940	-5.3%
January 6, 1992 - February 3, 1992	20.0 (35.95)	.4870	.4886	-0.3%
December 2, 1991 - February 3, 1992	18.61 (33.50)	.4827	.4913	1.8%
December 3, 1990 - February 4, 1991	18.84 (33.91)	.4799	.4940	2.9%

* $a/K \, (\Delta T)$

Z-SHAPE FACTOR METHOD (INSULATED SIDE)

The insulated side of the RTRA below grade is composed of 5 cm (2 in.) of extruded polystyrene, R-1.85 m^2K/W @ 24°C (R-10.5 @ 75°F) to a depth of .41 m for the heating period from January 6, 1992, to February 2, 1992. The Z-shape factor is extended to the insulated side. The U-values, however, do change. The new U-factors for the inside and outside become

Step 2

$$U_1 = [1/(4.28\ W/m^2/°K) + 0.41\ m/(1.81\ W/m/°K)]^{-1} = 2.17\ W/m^2°K,$$

$$U_2 = [1/(12.87\ W/m/°K) + 0.41\ m/(1.81\ W/m/°K)]^{-1} = 3.29\ W/m^2°K.$$

Using the modified U-values and steps 3 through 5 yields

Step 3

Bi = 1.46.

Step 4

r = 1.00.

Step 5

Z = 6.94.

The G^* and H^* parameters remain the same as for the uninsulated side.

The nondimensional heat flux using Step 6 is:

Step 6

S = 0.260.

The half-slab heat flux is

$$Q = k\ x\ \Delta T\ x\ S,$$

$$Q = 1.81(22.88-2.92)(0.260),$$

$$Q = 9.39\ \text{W/m}\ (9.78\ \text{Btuh/ft}).$$

The nondimensional heat flux is estimated using the Z-shape factor method for each of the seven heating periods listed in Table 2. The average percent reduction in heat loss estimated by this method, as shown in Table 2, varies from 45% to 47%.

Repeating the calculations for the experimentally derived heat flow mentioned above yields a nondimensional heat loss of 0.239 for the period December 2, 1991, to February 3, 1992. The experimental nondimensional heat loss for the period from December 3, 1991, to January 27, 1992, is 0.282. The floor heat load for the period from January 6, 1992, to February 3, 1992, is 8.63 W/m (8.98 Btuh/ft).

The heat loss at a point through the vertical insulation, shown in Figure 5, at location Y is

$$Q(Y) = \frac{1}{R_{th}}(T - T_2)$$

$$= \frac{T_1 - T_2}{R_{th}} \cdot \frac{T - T_2}{T_1 - T_2}$$

$$= \frac{T_1 - T_2}{R_{th}} \cdot \Theta.$$

TABLE 2
RTRA Insulated Slab Percent Savings Compared to Uninsulated Slab to Z-Shape Factor Prediction

Period	Percent Savings* $\left(\frac{Q_{INS} - Q_{UI}}{Q_{INS}}\right)$	S Z-Shape factor
February 14, 1989 - April 17, 1989	46.2	.2658
October 30, 1989 - April 8, 1990	46.7	.2602
November 11, 1990 - March 24, 1991	45.2	.2759
December 2, 1991 - March 9, 1992	46.2	.2658
January 6, 1992 - February 3, 1992	46.7	.2602
December 2, 1991 - February 3, 1992	46.5	.2630
December 3, 1990 - February 4, 1991	46.2	.2658

*The Q_{INS} and Q_{UI} are calculated using the Z-Shape Factor Method.

TABLE 3
Heat Flux Data from Insulated Slab on RTRA compared with Z-Shape Factor Predictions

Period	Average Q at 8" depth W/m^2 (Btu/ft^2)		Q at 14" depth W/m^2 (Btu/ft^2)	
	Data	Z-Shape Factor	Data	Z-Shape Factor
February 14, 1989 - April 17, 1989	-3.09 (-.98)	-3.41 (-1.08)	-2.11 (-.67)	-3.155 (-1.0)
October 30, 1989 - April 8, 1990	-4.51 (-1.43)	-3.91 (-1.24)	-3.41 (-1.08)	-3.6 (-1.14)
November 11, 1990 - March 24, 1991	-5.05 (-1.6)	-4.95 (-1.57)	-4.57 (-1.45)	-4.57 (-1.45)
December 2, 1991 - March 9, 1992	-4.48 (-1.42)	-4.2 (-1.33)	-3.66 (-1.16)	-3.91 (-1.24)
January 6, 1992 - February 3, 1992	-5.14 (-1.63)	-4.8 (-1.52)	-4.29 (-1.36)	-4.48 (-1.42)
December 2, 1992 - February 3, 1992	-4.83 (-1.53)	-4.51 (-1.43)	-4.0 (-1.27)	-4.23 (-1.34)
December 3, 1990 - February 4, 1991	-5.36 (-1.7)	-5.17 (-1.64)	-4.92 (-1.56)	-4.8 (-1.52)

Substituting for Θ in Equation 25 yields

$$Q(Y) = \left[\frac{T_1 - T_2}{R_{th}}\right] \Theta'_1 \left[\frac{1}{\gamma} e^{-\gamma Y}\right]$$

$$\Theta'_1 = \frac{Q_1 c}{kab\ (T_1 - T_2)}$$

$$= \frac{2c}{a} S$$

from Equation 16 and boundary conditions for Equation 23. Finally,

$$Q(Y) = \left[\frac{T_1 - T_2}{R_{th}}\right] (2S) \left[\frac{c}{a}\right] \cdot \left[\frac{1}{\gamma} e^{-\gamma Y}\right].$$

For the RTRA period from January 6, 1992, to February 3, 1992, and at a depth of 14 in. below grade,

$$= \frac{19.97°F}{1.8°K/m^2/W} (2) \cdot (.26) \left[\frac{.41m}{2.44m}\right] \left[\frac{.851}{.1846}\right]$$

$$= 4.48 W/m^2 (1.42\ Btuh/ft^2)$$

where

$$\gamma = \sqrt{\frac{2c^2}{kaR_{th}}}.$$

A final cross-check on the shape factor method described in this paper is shown in Table 3 with a comparison of the measured heat flux at two depths below grade on the insulated vertical foundation wall and the Z-shape factor predictions. The columns labeled "Data" are the average hourly heat flux for each heating season period and can be compared with the adjacent column labeled "Shape Factor." The largest difference occurs for the period from February 14, 1989, to April 17, 1989, which is expected because the insulation was installed in January, resulting in unsteady periodic conditions for the second half of the 1988-89 heating season.The average percent difference of the model compared to the data for the last three heating seasons—1989-90, 1990-91, and 1991-92—was −6.4% and 3.4% at the 8-in. and 14-in. depths. Overall, the model's average prediction is only 1.5% less than the measured heat flux. Figure 8 is a bar chart showing the average heat flux data at 8 inches and 14 inches below grade on the insulated slab foundation compared to the model predictions at these discrete points.

CONCLUSION

A new shape-factor method is introduced in this paper, which did a good job of predicting heat loss over four heating seasons from a well-characterized test building with an uninsulated slab and one with insulation. The model predicts overall heat loss from the uninsulated slab within 10% of measured response and, on average, predicts heat loss at discrete points on the insulated slab within ±7%. Extensive field measurements of an insulated and an uninsulated slab-on-grade foundation are used to provide measured inputs to this model and to provide validation to the model predictions. At this time, the procedure is only capable of providing heat load savings predictions of slabs insulated with vertical insulation.

NOMENCLATURE

Bi = Biot number (UL/k)
D = soil depth at which the temperature = T_3
ER = soil effective resistance
F = edge factor
G^* = ground flux parameter
H = total flux penetration distance (m)
H^* = horizontal flux penetration ratio
L = slab length (m)

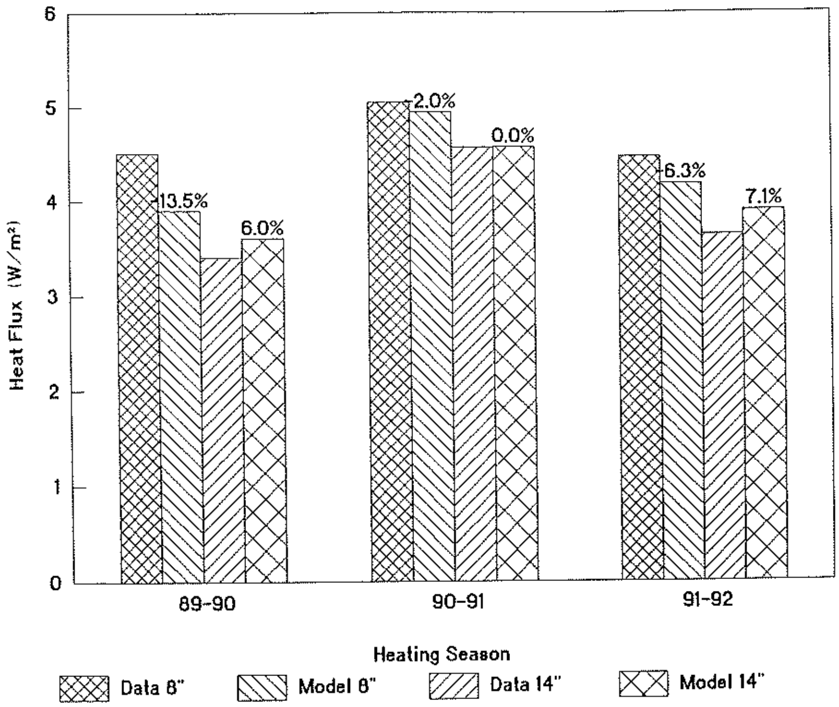

Figure 8 *Heat flux data on slab with vertical insulation at 8 in. and 14 in. below grade.*

Q	=	total heat transfer from a rectangular slab, normal to the surface, into the earth (W)
Q_a	=	total heat transfer from a rectangular slab, normal to the surface, in "a" direction (W)
Q_b	=	total heat transfer from a rectangular slab, normal to the surface, in "b" direction (W)
R	=	slab system resistance ($m^2 \cdot °C/W$)
R_{th}	=	R-value of the vertical slab insulation
S	=	shape factor or nondimensional heat loss component
T	=	temperature (°C)
U	=	overall heat transfer coefficient
a	=	horizontal length of slab (m)
b	=	horizontal width of slab (m)
c	=	insulation depth below grade (m)
h	=	heat transfer or film coefficient ($W/m^{2°} \cdot C$)
k	=	soil thermal conductivity ($W/m^{°} \cdot C$)
k_I	=	insulation thermal conductivity
t	=	insulation thickness

Subscripts

1	=	inside
2	=	outside
3	=	ground
th	=	thermal
i,j	=	indices

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